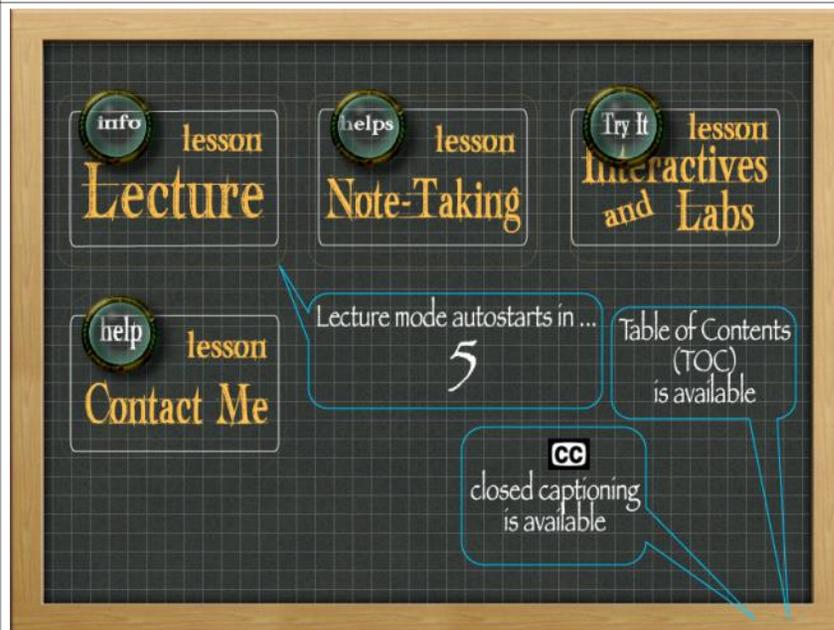


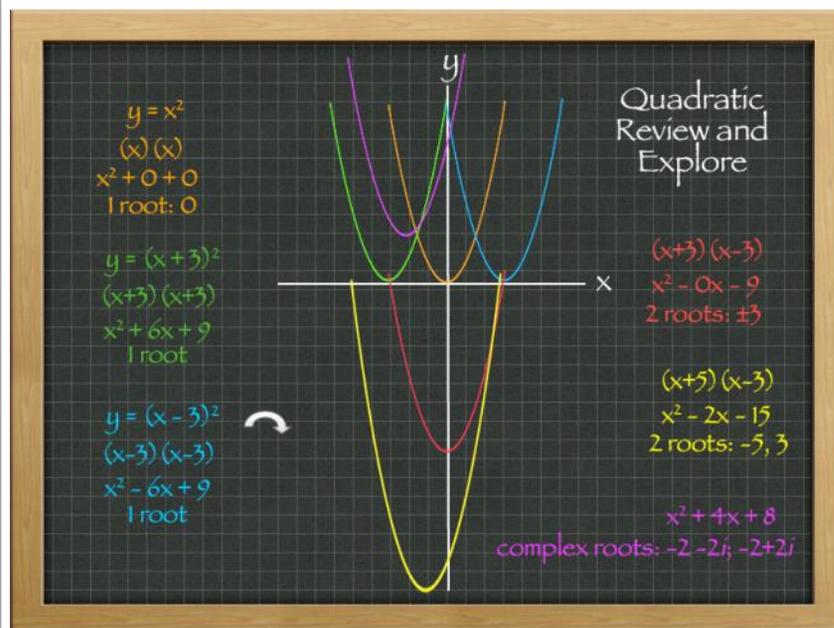
Quadratic Formula

Thursday, January 19, 2012
5:20 PM

Slide



Notes



I found that I understood quadratics the best once I started understanding them graphically, so I try to touch base with the graphical side of these periodically as I present the lessons to you.

When experimental or geometric data is plotted and it forms the shape of a parabola, you will find that a quadratic equation will be underlying the relations between the variables.

When it comes to solving for the roots of a quadratic equation, what you are finding is where the parabola crosses the x-axis. In later lessons you will be looking for the vertex which is the highest or lowest point depending on if it is 'smiling' or 'frowning'. Where the parabola crosses the y-axis can be worthy of calculating too.

For now though, lets focus on the roots. In the first three equations, notice how they all touch the x-axis in one place. The orange one is right at the 0,0 on the graph because nothing is added or subtracted from the x-squared. The green one has a +3 added to the x and notice

that it is on the x-axis at 03. It is the same number but the sign is reversed. You see that in the blue one too.

Let's play around with shifting the parabola along the y-axis. I will use the same equation as the blue one but make one of the three have a positive and the other one a negative sign. If you always use the same number but one is positive and the other negative that number will be where the parabola will cross the x-axis. That is because the two numbers cancel themselves out at the addition step when you do FOIL.

The last one has no roots. Notice when it is graphed it is above the x-axis. It has no real roots.

$ax^2 + bx + c = 0$ General Form of the Quadratic Equation
 ↓ solve for x
memorize!
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula:
 It is derived from completing the square, but takes less algebraic manipulation.

You have been calculating using completing the squares method to find the roots. That can be a bit of a tedious process, but it is worth learning because it works on all quadratics. However, some bright math students may have already thought to just go from the general formula and just create a formula to solve for x without having to do all that algebraic manipulation every time they need the roots. On the next slide, you will be able to follow the steps of how you can derive a formula to solve for x, but my tip to you is to just memorize it and it is there for you any time you need it. This is called the quadratic formula.

Completing the Square
 divide the 'a' lead coefficient off
 simplify
 begin isolating the x variable by moving the c/a off to the other side
 $\left(\frac{b}{a} \cdot \frac{1}{2}\right)^2 = \frac{b^2}{4a^2}$

$$\frac{ax^2 + bx + c = 0}{a \quad a \quad a \quad a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$- \frac{c}{a} \quad - \frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

Study this slide on how completing the squares was used to derive the formula. From time to time in this course and in future ones, you will be asked to derive the quadratic formula. It will be handy to review this slide to be sure you would be able to do it is called upon.

Completing the Square
divide the 'a' lead coefficient off

$$\frac{ax^2 + bx + c}{a} = \frac{0}{a}$$

simplify

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

begin isolating the x variable by moving the c/a off to the other side

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(\frac{b}{a} \cdot \frac{1}{2}\right)^2 = \frac{b^2}{4a^2}$$

simplify each side

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

square both sides and finish solving for x

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

time in this course and in future ones, you will be asked to derive the quadratic formula. It will be handy to review this slide to be sure you would be able to do it is called upon.

Use the quadratic formula to solve for the roots of the equation
 $3x^2 - 2x + 5 = 0$

$$ax^2 + bx + c = 0$$

substitute in the values for a, b, and c

$$x = \frac{-(-2) \pm \sqrt{((-2)^2 - 4(3)(5))}}{2(3)}$$

simplify

$$x = \frac{-(-2) \pm \sqrt{(-56)}}{6}$$

simplify: 2 can go in to the numerator and denominator. Get rid of the negative via an imaginary number

$$x = \frac{1 \pm \sqrt{14}i}{3}$$

rewrite in more useful format

$$x = \frac{1}{3} \pm \frac{\sqrt{14}}{3}i$$

Most of the time though, you will just be able to use the formula. Let's see how to use it.

We will use the quadratic formula to solve for the roots of the equation $3x^2 - 2x + 5 = 0$. Minus $2x$ plus 5 is equal to 0.

First jot down the quadratic formula and use the general form of a quadratic equation to see what a, b, and c will be.

Then just substitute in the values.

Simplify.

You can pull a 2 out of the numerator so that it can cancel a 2 that can be pulled out of the 6 in the denominator. You can also take care of the negative under the root bar. That will make it $14i$. The imaginary number popping up here may alert you to the high probability that this equation may end up being above the x-axis when it is graphed and therefore have no roots.

We will rewrite it to a more useful format to be sure. Confirmed, this one will be positioned above about $1/3$ on the x-axis, but it will never touch it.

Memorizing the Quadratic Formula is your primary task in this lesson. Let's see how well you have it already ...

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$$x = \frac{\square \pm \sqrt{(b^2 - \square)}}{\square}$$

Submit

Congratulations!
You have completed
this topic

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this lesson if you wish...

